Grasp Quality Evaluation in Underactuated Robotic Hands

Maria Pozzi[1], Ashok M. Sundaram[2,3], Monica Malvezzi[1], Domenico Prattichizzo[1], Máximo A. Roa[2]

***Abstract*— Underactuated and synergy-driven hands are gaining attention in the grasping community mainly due to their simple kinematics, intrinsic compliance and versatility for grasping objects even in non structured scenarios. The evaluation of the grasping capabilities of such hands is a challenging task. This paper revisits some traditional quality measures developed for multi-fingered, fully actuated hands, and applies them to the case of underactuated hands. The extension of quality metrics for synergy-driven hands for the case of underactuated grasping is also presented. The performance of both types of measures is evaluated with simulated examples, concluding with a comparative discussion of their main features.**

I. Introduction

Multi-fingered hands have been developed and studied over several decades. However, applications of such hands to environments out of research labs have been scarce, mainly due to their control complexity, lack of robustness, and high cost. Underactuated and passively compliant hands are nowadays becoming increasingly popular because of their intrinsic compliance, simple kinematics, and high grasping performance even in unstructured environments [1], [2]. New hand designs have also profited from the concept of synergies, i.e., reduced dimensionality for controlling the motion of multiple degrees of freedom in hands [3].

Measuring the performance of this new type of hands is a challenging task. For traditional grasp approaches on fully actuated multi-fingered hands, where the contact points on the object and a corresponding pose of the hand are computed, the concepts of form closure and force closure [4] have been extensively exploited for analytic derivation of grasp strategies. As multiple grasps can fulfill these properties, an optimal grasp is chosen according to some quality measure, i.e., an index that quantifies the goodness of a grasp. Different grasp quality measures have been proposed for grasp analysis, based either on the geometrical location of the contact points or on the hand configuration; combination of measures from the two groups are also possible through weighted sums of different quality criteria [5]. Although there have been initial attempts for directly applying these measures for the case of underactuated hands [6], this application is not straightforward due to the nature of underactuated grasping: the mechanical compliance of the hand takes care of adapting the hand shape to better suit the object, thus finding specific contact points or precomputing hand configurations is not relevant anymore. Evaluations on simulation have also been used as indicators of grasping performance in these hands, for instance maximizing the region of acquisition of the object or the number of contacts with the hand based on a certain distribution of object locations within a given range [7], [8]. Also, experimental evaluation of resistance to specific external disturbances has been considered as a meaningful way to measure the grasp robustness when underactuation is present [1], [2], [9].

Classical quality measures, developed for fingertip grasps, could in principle be used for analyzing power grasps, although the high number of contact points would make the computations more expensive [10]. The explicit consideration of the limited forces that some parts of the hand can apply on the object facilitates the definition of contact robustness, i.e., how far a contact is from violating contact constraints, which is different from grasp robustness, i.e., how far the grasp is from overcoming the object immobilization constraint [11]. Thus, the effective capacity of the robotic hand to apply contact forces can be used for defining two grasp quality measures, the Potential Contact Robustness (PCR), and the Potential Grasp Robustness (PGR) [4], [11]. These measures are explicitly based on the computation of the subspace of controllable internal forces, which depends on the structure and type of actuation of the hand and on the location of the contact points on the object. These characteristics suggest that, at least from a theoretical point of view, PCR and PGR can be especially suitable for evaluating grasping in underactuated hands, as proposed in this paper. Also, the paper presents the extension of these two indices for hands actuated by soft synergies, which is a very general, realistic, and versatile way to model underactuation [12].

To complement the analysis of grasp quality in underactuated hands, this paper also extends for this case the quality measure that computes the largest-minimum resisted wrench in any direction, based on the analysis of the grasp wrench space [13]. This is one of the most extended quality measures in literature [14], and originally has two different formulations, depending on whether the fingers are powered by a single actuator or by independent actuators. Due to computational reasons, most works so far implicitly use the assumption that the hand is powered by a single actuator

*This work was supported by the European Comission Grant No. H2020-ICT-645599, "SOMA": SOft MAnipulation

[1] M. Pozzi, M. Malvezzi, and D. Prattichizzo are with the Department of Information Engineering and Mathematical Science, University of Siena, Siena, Italy and also with the Department of Advanced Robotics, Istituto Italiano di Tecnologia, Genova, Italy. Email: {pozzi, malvezzi, prattichizzo}@diism.unisi.it

[2]A.M. Sundaram and M.A. Roa are with the Institute of Robotics and Mechatronics, German Aerospace Center (DLR), Wessling, Germany. Email: {ashok.meenakshisundaram, maximo.roa}@dlr.de.

[3]A.M. Sundaram is also with the Department of Computer Science, Bonn-Rhein-Sieg University of Applied Sciences, Sankt Augustin, Germany.

and that the forces applied at the contact points are unitary (which requires a simple convex hull computation in the wrench space, as described later in detail), thus neglecting the influence of the hand on the grasp quality computation [5]. This leads to a measure that does not enclose a physically meaningful quantity, and that cannot be used to compare grasp robustness among different objects. The quality measures based on wrench space analysis are reconsidered in this paper, revising their assumptions in order to obtain a meaningful measure that can also be extended for the analysis of underactuated hands. The measures based on the wrench space and based on the contact and grasp robustness are compared in simulated examples, to gain insights into the different aspects that the measures cover for the analysis of underactuated grasping.

The rest of the paper is organized as follows. Section II provides a brief mathematical background on grasping. Section III analyzes the measures based on computations on the wrench space, including their application to underactuated hands. Section IV considers the measures of contact and grasp robustness, and their extension to the analysis of underactuation with synergies. Section V presents the application of the measures to analysis of precision and power grasps with an anthropomorphic hand in the cases of full actuation and underactuation. Section VI concludes the paper.

II. Mathematical Background: Grasping

Consider a robotic hand grasping an object in a static equilibrium. Let $\{\mathbf{N}\}$ be a fixed reference frame, and $\{\mathbf{B}\}$ be the reference frame attached to the grasped rigid object in 3D space, usually at its center of mass. All vectors are expressed in $\{\mathbf{N}\}$, except where otherwise indicated. For a hand with rigid links and n_q revolute joints, the vectors $\mathbf{q} = \left[q_1, q_2, \ldots, q_{n_q}\right]^T \in \mathbb{R}^{n_q}$ and $\boldsymbol{\tau} = \left[\tau_1, \tau_2, \ldots, \tau_{n_q}\right]^T \in \mathbb{R}^{n_q}$ represent the joint angles and joint torques, respectively. The object pose and twist with respect to $\{\mathbf{N}\}$ are indicated with $\mathbf{u} \in \mathbb{R}^6$ and $\boldsymbol{\nu} \in \mathbb{R}^6$, respectively. The external wrench applied to the object is given by $\mathbf{w} = \left[\mathbf{F}^T \; \mathbf{m}^T\right]^T \in \mathbb{R}^6$, where $\mathbf{F} \in \mathbb{R}^3$ and $\mathbf{m} \in \mathbb{R}^3$ are the force and torque acting on the object, the torque being computed with respect to the origin of $\{\mathbf{B}\}$.

Let us suppose that the hand contacts the object at n_c points, modeled as hard finger contacts (point contact with friction) [15]. At each contact point $\mathbf{c}_i$, a force $\mathbf{f}_i = \left[f_{i,n}, f_{i,t_1}, f_{i,t_2}\right]^T \in \mathbb{R}^3$ expressed in the contact reference frame $\{\mathbf{C}\}_i$, is applied on the object; the normal component is given by $f_{i,n}$, while f_{i,t_1} and f_{i,t_2} are the tangential components of the force. In order to prevent detachment and slippage at the i-th contact, the force must satisfy the positivity constraint (1) and Coulomb's friction constraint (2),

$$f_{i,n} \geq 0 \tag{1}$$

$$\sqrt{f_{i,t_1}^2 + f_{i,t_2}^2} \leq \mu_i f_{i,n} \tag{2}$$

where μ_i is the static friction coefficient at the contact i.

Additional constraints on the forces can be considered, such as upper bounds on the magnitude of the i-th contact force (3) or on the sum of all contact forces (4)

$$\| \mathbf{f}_i \| \leq f_{i,max} \quad \text{with} \quad f_{i,max} \geq 0 \tag{3}$$

$$\sum_{i=1}^{n_c} \| \mathbf{f}_i \| \leq f_{tot,max} \quad \text{with} \quad f_{tot,max} \geq 0 \tag{4}$$

Let $\mathbf{f}$ be the vector of stacked contact forces, $\left[\mathbf{f}_1^T, \mathbf{f}_2^T, \ldots, \mathbf{f}_{n_c}^T\right]^T \in \mathbb{R}^{3n_c}$. The static equilibrium of the object and of the hand is described by (5) and (6), respectively,

$$\mathbf{w} = -\,\mathbf{G}\mathbf{f} \tag{5}$$

$$\boldsymbol{\tau} = \mathbf{J}^T \mathbf{f} \tag{6}$$

where $\mathbf{G} \in \mathbb{R}^{6 \times 3n_c}$ is the Grasp matrix, and $\mathbf{J} \in \mathbb{R}^{3n_c \times n_q}$ is the hand Jacobian matrix [15]. The contribution $\mathbf{w}_i$ of each contact force $\mathbf{f}_i$ to the equilibrium equation (5) is given by $\mathbf{w}_i = \mathbf{G}_i \mathbf{f}_i$, where $\mathbf{G}_i$ is a matrix that maps the force $\mathbf{f}_i$ expressed in $\{\mathbf{C}\}_i$ to the object frame $\{\mathbf{B}\}$.

III. Measures Based on Wrench Space

The analysis of the Grasp Wrench Space (GWS) has been traditionally used to define different quality measures [5]. Among them, a widely used criterion is the computation of the largest minimum resisted wrench (Q_ϵ) [13]; it represents the maximum perturbation wrench that a grasp can resist in any direction. This section revises the different ways of constructing the GWS and the corresponding Q_ϵ measures, and proposes a method for computing a physically meaningful measure for the case of underactuated hands.

A. Largest-minimum Resisted Wrench (Q_ϵ)

The GWS contains the set of all wrenches that can be applied on the object through the contact points. To obtain those wrenches, the friction cone describing the contact forces that satisfy (1) and (2) is usually approximated with a polygonal pyramid of m edges. For a given contact i, the wrenches generated by the discretized friction cone, or primitive wrenches, are grouped in a wrench set $\mathbf{w}_i^{d,c} = [\mathbf{w}_{i,1}^T, \mathbf{w}_{i,2}^T, \ldots, \mathbf{w}_{i,m}^T]$. The set of primitive wrenches for a given grasp with n_c contacts is then $\mathcal{W}^c = [\mathbf{w}_1^{d,c}, \mathbf{w}_2^{d,c}, \ldots, \mathbf{w}_{n_c}^{d,c}]$.

The boundaries of the GWS can be constructed by

$$GWS = CH\,(\mathcal{P}) \tag{7}$$

where CH denotes the convex hull operation over a set $\mathcal{P}$ computed from the primitive wrenches, according to certain assumptions on the forces at the fingers. Using normalized contact forces, two ways of constructing the set $\mathcal{P}$ were proposed in [13], as discussed below. The Q_ϵ measure is then calculated as the radius of the largest ball centered at the origin and fully contained in the GWS,

$$Q_\epsilon = \min_{w \in GWS} \| w \| \tag{8}$$

1) Limiting the sum of all contact forces: In this case, the sum of modules of the forces applied by all the contacts has an upper limit, unitary in general (i.e., $f_{tot,max} = 1$ in (4)). Then, the corresponding GWS_S can be constructed as in (7) with the set $\mathcal{P} = \mathcal{W}^c$, and the corresponding quality measure from (8) is denoted by Q_{ϵ_S}. The described assumption is physically interpreted as having a limited power source (actuator) for all the fingers, which is not the case in most designs of multi-fingered hands. However, the simplicity and fast computation of GWS_S under this assumption have made it the most common method for computing grasp quality [5].

2) Limiting the individual contact forces: In this case, each contact force has an upper limit, unitary in general (i.e., $f_{i,max} = 1$ in (3)). The corresponding GWS_F can be constructed as in (7), with the set $\mathcal{P}$ given by the union of the Minkowski sum of all the possible combinations C of individual contact wrenches:

$$\mathcal{P} = \bigcup_{i=1}^{n_c} \left(\bigcup_{j=1}^{{}^iC_{n_c}} \left(\bigoplus \mathcal{W}_j^c \right) \right) \quad \text{with} \quad \mathcal{W}_j^c \in \binom{\mathcal{W}^c}{i} \tag{9}$$

where $\mathcal{W}_j^c$ represents the j^{th} combination set of $\mathcal{W}^c$ in the i^{th} iteration (i.e. taking i elements from $\mathcal{W}^c$). The corresponding measure calculated from (8) is denoted as Q_{ϵ_F}.

Physically, this corresponds to a limited independent power source for each finger. Note that the individual contributions of different contact forces, as well as their possible combinations, are considered in (9). This is a more realistic assumption in the case of multifingered hands [16]. However, due to the Minkowski sum operation, there are a total of $(m+1)^{n_c}$ elements for the CH operation. The complexity then increases in the order of $O(m^{n_c})$, while the complexity for the previous case (union operation) is in the order of $O(mn_c)$. As the CH computation will explode for large number of contacts, GWS_F is rarely used despite being physically more relevant. To decrease the computational time as much as possible for the computation of this measure, an incremental construction of the grasp wrench space inspired by [17] is used in this work. Basically, it uses few primitive wrenches at the beginning, and incrementally refines the grasp wrench space computation only on the weakest directions, which contribute to the Q_ϵ measure. Additionally, note that the quality metrics defined in the wrench space combine quantities for forces and torques; to obtain a consistent metric, a suitable scaling factor for the torque components can be defined, as discussed in detail in [18].

B. Considerations for Underactuated Grasping

It is now clear that GWS_S is relevant for hands with a single actuator and GWS_F is relevant for hands with multiple individual actuators per contact/finger. In the case of multifingered underactuated hands, there can be several contacts that are controlled through a single actuator, or contacts that are individually controlled by one actuator. To consider this combination in the computation of GWS, we define an actuation matrix $\mathbf{A} \in \mathbb{R}^{n_a \times n_c}$ for each grasp, with n_a being the number of actuators. For each row, i.e. one actuator, the columns of the contacts that are controlled through that actuator are filled with a value of one, while all other columns are filled with zeros to indicate that the actuator has no direct influence on those contacts. This simplification does not consider differences in force transmission ratios for each link, but rather considers only direct influence of an actuator on a given contact point. Some examples are provided in Section III-D. By using the information stored in this matrix, the combination of discretized wrenches affected by the same actuator is given by

$$\mathbf{w}_i^{d,a} = \bigcup_{j=1}^{n_c} \begin{cases} \mathbf{w}_j^{d,c} & \text{if } \mathbf{A}_{i,j} = 1 \\ empty & \text{otherwise} \end{cases} \tag{10}$$

All the actuator-wise discretized contact wrenches are collected in the set $\mathcal{W}^a = [\mathbf{w}_1^{d,a}, \mathbf{w}_2^{d,a}, \ldots, \mathbf{w}_{n_c}^{d,a}]$. The corresponding GWS_U can be constructed as in (7), with the set $\mathcal{P}$ given by

$$\mathcal{P} = \bigcup_{i=1}^{n_a} \left(\bigcup_{j=1}^{{}^iC_{n_a}} \left(\bigoplus \mathcal{W}_j^a \right) \right) \quad \text{with} \quad \mathcal{W}_j^a \in \binom{\mathcal{W}^a}{i} \tag{11}$$

Note that equations (11) and (9) are almost the same, except that (9) uses $\mathcal{W}^c$ consisting of discretized contact wrenches while (11) uses $\mathcal{W}^a$ consisting of actuator-wise discretized contact wrenches. The corresponding quality measure calculated for GWS_U is denoted by Q_{ϵ_U}.

C. Realizable Forces

Traditionally, it is assumed that each contact can apply a unitary normal force [13]. However, to obtain a realistic quality measure the magnitude of the force that is physically realizable has to be directly considered for the GWS construction [19]–[21].

The maximum contact forces realizable by each finger can be derived as follows. For each contact i, $\mathfrak{f}_i$ represents the unit contact normal force $\mathbf{f}_i$ in the finger base coordinate frame. Let $\mathfrak{F}_j$ be the set of all the unit contact normal forces generated by the finger j and expressed in the finger base coordinate frame. The finger torques $\boldsymbol{\tau}_j$ required to achieve $\mathfrak{F}_j$ are calculated with the joint configuration $\mathbf{q}_j$ and the corresponding body Jacobian $\mathbf{J}(\mathbf{q}_j)$ as

$$\boldsymbol{\tau}_j = \mathbf{J}(\mathbf{q}_j)^T . \mathfrak{F}_j \tag{12}$$

The resulting $\boldsymbol{\tau}_j$ is scaled by a factor k, until one of the joint torques reaches its corresponding torque limit. The maximum realizable normal force at the contact i is then given by $k\mathfrak{f}_i$. By using this realizable force for each contact, the quality measure will represent the realizable physical magnitude of wrench that can be counteracted by the grasp in any direction. Note that the maximum contact force calculated in this way is the physical limit of the force, but does not represent the real force applied at each contact in a particular grasp situation.

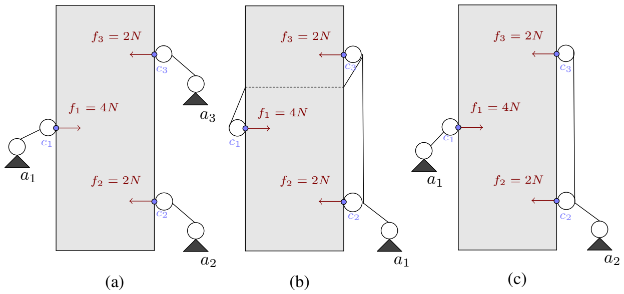


Fig. 1: Grasping a rectangular object with three contacts with the same normal forces and hand-object contact configuration, and using: (a) a fully actuated hand; (b) an underactuated hand using a single actuator; (c) an underactuated hand using two actuators.

D. Comparison

To illustrate the differences in the three approaches for constructing GWS, we consider as an example a given grasp on a 2D object, obtained with three different hands with different mechanisms of actuation, as shown in Fig. 1 The grasp in Fig. 1a is fully actuated with three actuators, while the grasps in Fig. 1b and Fig. 1c are underactuated with one and two actuators, respectively. Actuation matrices for the three cases are, respectively,

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}; \mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

Even if the same normal forces can be applied at the contacts in the three cases, the ability of the hand to apply these forces completely differs from one another. A typical grasp analysis will lead to the GWS_S shown in Fig. 2b, with corresponding quality Q_{ϵ_S}, for the three grasps in Fig. 1. Actually, this measure is appropriate for the grasp in Fig. 1b, but it underestimates the ability of grasps in Fig. 1a and Fig. 1c. A fully actuated case, with an independent actuator per contact/finger, is depicted in Fig. 1a; the corresponding GWS_F is shown Fig. 2a, and the quality is Q_{ϵ_F}. This measure, however, would overestimate the ability of grasps in Fig. 1b and Fig. 1c. With a suitable definition of the actuation matrix (Section III-B), all the cases can be properly analyzed.

IV. Measures of Contact and Grasp Robustness

The Potential Contact Robustness (PCR) and the Potential Grasp Robustness (PGR) are two grasp quality indexes that depend on the hand configuration and on the position of the contact points on the object. The PCR was firstly introduced in [4] and was compared to the PGR in [11].

A. Assumptions and Definitions

Before defining the indexes we must add further details to the grasp model. In particular, let us assume that the contacts and the joints are not perfectly rigid and their stiffness can be linearly modeled with $\mathbf{K}_s \in \mathbb{R}^{3n_c \times 3n_c}$, the contact stiffness matrix, and $\mathbf{K}_p \in \mathbb{R}^{n_q \times n_q}$, the joint stiffness matrix [22]. Under these hypotheses, we can relate the variations of the grasp variables w.r.t. an equilibrium reference configuration with eq. (13) and (14) [23].

$$\Delta\mathbf{f} = \mathbf{K}_s(\mathbf{J}\Delta\mathbf{q} - \mathbf{G}^T\Delta\mathbf{u}) \quad (13)$$

$$\Delta\boldsymbol{\tau} = \mathbf{K}_p(\Delta\mathbf{q}_r - \Delta\mathbf{q}) \quad (14)$$

$\Delta\mathbf{q}_r$ indicates a variation of the reference joint configuration.

Under the additional hypotheses that the grasp is not indeterminate, i.e. $\mathcal{N}(\mathbf{G}^T) = \mathbf{0}^{(1)}$ [23], and that the contact points do not change by rolling, let us define the vector:

$$\mathbf{d}(\mathbf{f}) = [d_{1,c}, d_{1,f}, d_{1,max} \ldots, d_{n_c,c}, d_{n_c,f}, d_{n_c,max}] \quad (15)$$

where $d_{i,c}$ is the normal component of the i^{th} contact force ($d_{i,c} = f_{i,n}$), $d_{i,f}$ is the distance of $\mathbf{f}_i$ from the friction cone defined by (1) and $d_{i,max} = f_{i,max} - \parallel \mathbf{f}_i \parallel$. The vector $\mathbf{d}(\mathbf{f})$ records how far the grasp is from violating the friction constraints and the maximum force limits. Let us define the quantity $d_{min} \in \mathbb{R}$ as the minimum element of vector $\mathbf{d}(\mathbf{f})$. Then (16) is a sufficient condition for having a contact force perturbation $\Delta\mathbf{f}$ such that the constraints (1)-(3) are not violated:

$$\parallel \Delta\mathbf{f} \parallel \leq d_{min} \quad (16)$$

In order to find a measure of contact robustness, we have to express eq. (16) as a constraint on the external disturbances $\Delta\mathbf{w}$ acting on the object, as in (17).

$$\parallel \Delta\mathbf{w} \parallel \leq \frac{d_{min}}{\sigma_{max}(\mathbf{G}_K^R)} \quad (17)$$

In (17), $\mathbf{G}_K^R = \mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)$ is the weighted right pseudoinverse of $\mathbf{G}$, $\mathbf{K} = (\mathbf{K}_s^{-1} + \mathbf{J}\mathbf{K}_p^{-1}\mathbf{J}^T)^{-1}$ is the grasp stiffness matrix, and $\sigma_{max}(\mathbf{G}_K^R)$ is the maximum singular value of $\mathbf{G}_K^R$. Computations needed to pass from (16) to (17) are explained in [11] and are not detailed here for the sake of brevity. The right side of eq. (17) represents a measure of contact robustness: the bigger is $d_{min}/\sigma_{max}(\mathbf{G}_K^R)$, the bigger is the $\Delta\mathbf{w}$ compatible with the friction constraints, the more robust is the grasp.

According to [24] the controllable contact forces can be expressed as $\mathbf{f} = -\mathbf{G}_K^R\mathbf{w} + \mathbf{E}\mathbf{y}$, where $\mathbf{E}$ is a basis for the subspace $\mathcal{F}_a$ of the controllable internal forces and $\mathbf{y}$ is a vector that parameterizes that subspace. In order to consider in our computations only the controllable contact forces, we can define the quantity $d_{min}^{\mathcal{F}_a} = \min\{\mathbf{d}(\mathbf{G}_K^R\mathbf{w} + \mathbf{E}\mathbf{y})\}$.

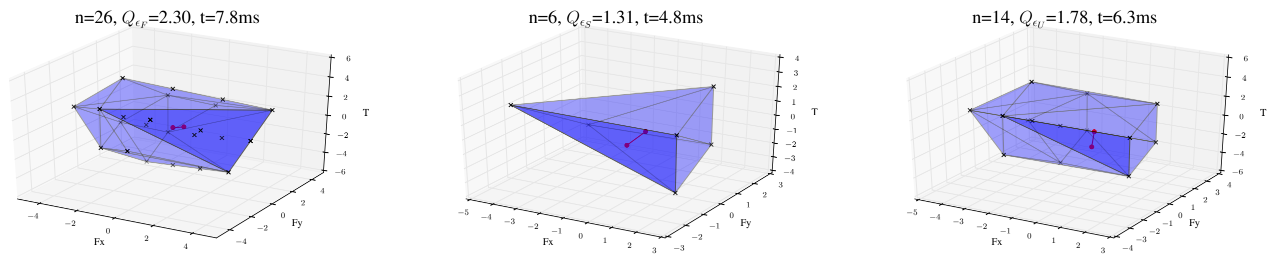
The Potential Contact Robustness (PCR) can be then defined as follows:

$$\text{PCR} = \max_{\mathbf{y}} \frac{d_{min}^{\mathcal{F}_a}}{\sigma_{max}(G_K^R)}, \quad (18)$$

where $d_{min}^{\mathcal{F}_a} = \min\{\mathbf{d}(\mathbf{G}_K^R\mathbf{w} + \mathbf{E}\mathbf{y})\}$ is a function of $\mathbf{y}$.

Let $\tilde{\mathbf{y}} = \text{argmax}\left(d_{min}^{\mathcal{F}_a}/\sigma_{max}(\mathbf{G}_K^R)\right)$, then any disturbance wrench $\Delta\mathbf{w}$ such that $\parallel \Delta\mathbf{w} \parallel \leq$ PCR can be resisted without any slippage or detachment, provided that the internal force $\mathbf{E}\tilde{\mathbf{y}}$ is actuated.

(1) $\mathcal{N}(\mathbf{A})$ indicates the null space of matrix $\mathbf{A}$.



(a) (b) (c)

Fig. 2: Different computations of GWS for the grasps shown in Fig. 1, depending on the type of actuation. The number of wrench vectors n, quality Q_ϵ, and the time taken for computation t are also shown. The wrench vectors are denoted by $\times$. The weakest facet is denoted in dark blue, and the corresponding quality is represented with the red line. (a) GWS_F for the grasp in Fig. 1a (b) GWS_S for the grasp in Fig. 1b (c) GWS_U for the grasp in Fig. 1c

In grasps with many contact points (e.g. power grasps), the grasp is stable and can resist effectively external wrenches even if some contacts are detached or slipping, provided that there are enough contacts that are holding the object. In this case, however, the PCR might underestimate the grasp quality. The PGR index is an extension of the PCR that can be applied in these cases because it considers that a grasp can be stable even if some of the contacts do not satisfy the friction constraints (1), (2).

The hypothesis that we have to introduce to define the PGR is that the i^{th} contact, after the action of an external disturbance $\Delta\mathbf{w}$, can be in three different states:

- State 1: constraints in (1) and in (2) are both satisfied at the i^{th} contact. In this case the the contact force can be transmitted in any direction through the contact.
- State 2: only (1) is satisfied. In this case the contact force can be transmitted only in the normal direction to the contact.
- State 3: constraints in (1) and in (2) are both violated. In this case the contact is considered as detached.

Considering that each of the n_c contact points can be in 3 different states, there will be 3^{n_c} possible grasp combinations C_j. Each of them will have a certain global stiffness matrix $\mathbf{K}(C_j) = (\mathbf{K}_s^{-1} + \mathbf{J}\mathbf{K}_p^{-1}\mathbf{J}^T)^{-1}$, with $\mathbf{K}_s = \text{diag}(\mathbf{K}_{1,s}, \ldots, \mathbf{K}_{n_c,s})$, in which the dimensions of $\mathbf{K}_{i,s}$ depend on contact state, and its elements depend on contact surface properties [11], a certain basis for the subspace of controllable internal forces $\mathbf{E}(C_j)$, and a certain Potential Contact Robustness PCR(C_j).

The Potential Grasp Robustness (PGR) for a certain grasp with n_c contact points can be computed as in (19) and it maximizes the PCR index with respect to all the configurations C_j.

$$\text{PGR} = \max_{C_j, j=1,\ldots,n_c} \text{PCR}(C_j) \tag{19}$$

The PCR can be seen as a particular case of the PGR: if for a certain grasp the optimal combination $C_j^* = \text{argmax}(\text{PCR}(C_j))$ has all the contact points in State 1, the PGR and PCR coincide.

The approach used by Prattichizzo et al. [11] is mainly based on geometrical considerations that make the expression of the PGR very intuitive. To reduce the computational load when computing the PCR (and consequently the PGR), we use a suitable defined cost function that guarantees convergence of the problem of maximizing the distance from the friction cone [4].

B. Modeling the Underactuation

Thanks to their definitions, the PCR and the PGR can account for the underactuation of the hand. In this section, hands actuated by postural synergies are considered. In the field of neuroscience, Santello et al. in [25] demonstrated that most of the variance in human grasping configurations can be explained by few postural synergies, i.e. high level coordinated movements through which the human brain controls how the hand is shaped to grasp objects. Due to their characteristics, postural synergies can be exploited in order to control robotic hands, so to reduce their number of actuators without affecting their versatility. In [12] authors show how to compute the subspace of controllable internal forces in case of synergistic actuation. Its expression is written in (20), where $\mathbf{E}_s$ is a basis for $\mathcal{F}_{a,s}$, and $\mathbf{S} \in \mathbb{R}^{n_q \times n_z}$ is the Synergy Matrix that relates the joint reference velocity $\dot{\mathbf{q}}_r$ to the synergy vector $\mathbf{z} \in \mathbb{R}^{n_z}$ through the equation $\dot{\mathbf{q}}_r = \mathbf{S}\dot{\mathbf{z}}$ [26].

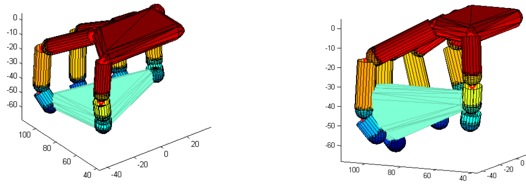
$$\mathcal{F}_{a,s} = \mathcal{R}^{(2)}(\mathbf{E}_s) = \mathcal{N}(\mathbf{G}) \cap \big(\mathcal{R}(\mathbf{KJS}) + \mathcal{R}(\mathbf{KG}^T)\big) \tag{20}$$

It is sufficient to replace $\mathcal{F}_a$ and $\mathbf{E}$ with $\mathcal{F}_{a,s}$ and $\mathbf{E}_s$ in the definitions of PCR and PGR to obtain the expression of the two measures for the case of synergy-actuated hands.

C. Evaluations

To compare the PCR and PGR measures we carried out simulations using SynGrasp, a MATLAB Toolbox that provides models and tools for grasp analysis with fully and under-actuated hands [27]. Fig. 4 shows the results obtained for a precision grasp done by an anthropomorphic hand model with 20 degrees of freedom (Fig. 3a) [28].

(2) $\mathcal{R}(\mathbf{A})$ indicates the column space of matrix $\mathbf{A}$.



(a) (b)

Fig. 3: Anthropomorphic hand grasping an object with 5 (Fig. 3a) and 6 (Fig. 3b) contact points.

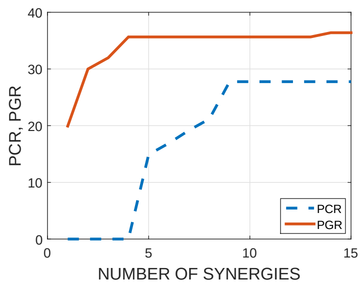


Fig. 4: PCR and PGR with respect to the number of activated synergies for a precision grasp with 5 contact points.

Both the PCR and the PGR are non decreasing with respect to the number of activated synergies n_z. Since n_z corresponds to the number of actuated degrees of freedom of the hand, the higher it is, the wider is the subspace of controllable internal contact forces $\mathcal{F}_{a,s}$. The larger is $\mathcal{F}_{a,s}$, the bigger is the space with respect to which the optimization procedures (18) and (19) search, which explains the trends in Fig. 4.

From Fig. 4 it can be seen that PGR is always greater than PCR, which is due to the definition of the PGR that, as underlined also in Section IV-A, is an extension of the PCR and, as such, is always greater or equal to it. In particular, it is interesting to notice that the PGR index, differently form the PCR, detects that there can be a force closure grasp (i.e. a grasp with PGR > 0) even when only one synergy is activated. This result suggests that the Potential Grasp Robustness (PGR) index is less conservative but more realistic than the PCR, and thus it is more suitable for detecting stable grasps, especially with underactuated hands.

Fig. 5 shows how the PGR index varies with respect to the number of activated synergies for different grasp configurations. Specifically, we consider precision grasps with 3 (thumb, index, middle), 4 (thumb, index, middle, ring), and 5 (thumb, index, middle, ring, little) contact points and a power grasp with 6 contacts located on the pads and distal phalanges of thumb, index and middle (Fig. 3b). It can be observed that the PGR index $i)$ grows with the number of contact points, $ii)$ is very similar in case of 4 and 5 contacts, $iii)$ improves considerably for the power grasp. The second characteristic quantitatively expresses the empirical evidence that adding the little finger to a precision grasp does not

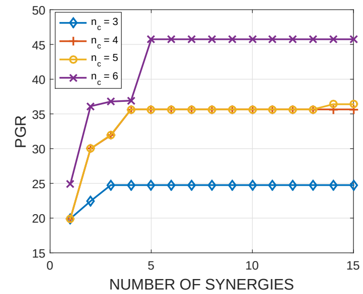


Fig. 5: Plot of the PGR index with respect to the number of activated synergies for 4 different types of grasp configurations.

substantially affect its stability.

V. Discussion

The two categories of grasp quality measures presented in this paper are based on the force closure property, but convey different interpretations about the grasp itself. The largest-minimum resisted wrench provides the physical magnitude of wrench that can be resisted in any direction. Because of its direct physical relevance in the wrench domain, this measure can be directly used to verify task requirements like, for instance, applied external wrenches on the object. The contact and grasp robustness compute how far a grasp is from violating contact constraints and from violating object immobilization constraints, respectively. They consider the effective capabilities of the hand to control internal forces, and therefore are more suitable in terms of controllability.

We choose to compute the PGR and Q_ϵ measures for three precision grasps with 3, 4, and 5 contacts (the same set of grasps used in Section IV-C), and a power grasp with 6 contact points. All the grasps are performed with an anthropomorphic hand model included in SynGrasp MATLAB Toolbox [27]. We evaluate the measures with respect to the static friction coefficient μ_i and the maximum applicable force $f_{i,max}$ at each contact. These two quantities strongly affect the success of a grasp, and their inclusion in the computation of the grasp quality is crucial.

The friction coefficient depends on which materials the hand and the object are made of, and plays a fundamental role when the grasped object is fragile or when the hand performs manipulation with sliding [29]. In the context of soft manipulation, having a quality measure that accounts for the friction coefficient is crucial, not only because soft materials have different mechanical properties, but also because soft hands will be used for grasping and manipulating delicate objects like food. The maximum realizable force at each contact based on the hand actuation and structure can be computed (Section III-C) [19], [20]; however, another parameter that should be accounted for when defining $f_{i,max}$ is the fragility of the object [30].

In this section, since we work on simulated hand and object without considering a specific robotic hand, we suppose that the friction coefficient and the maximum force are

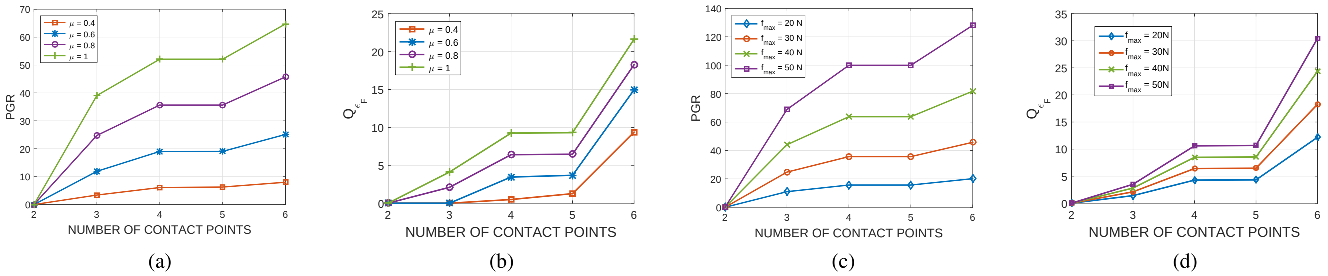


Fig. 6: (a), (b): PGR and Q_{ϵ_F} indices with respect to the number of contact points and the friction coefficient μ. The value of $\mathbf{f}_{max}$ is fixed to $30N$. (c), (d): PGR and Q_{ϵ_F} indices with respect to the number of contact points and the maximum applicable force at the contacts f_{max}. The value of μ is fixed to 0.8.

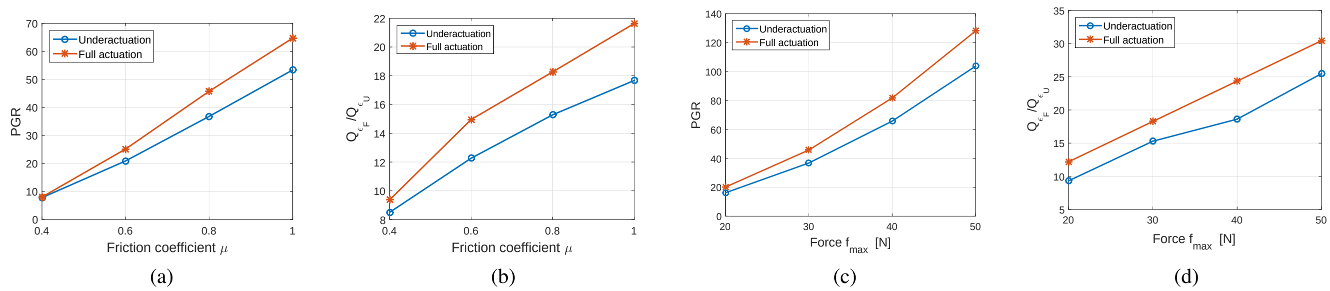


Fig. 7: (a), (b): PGR and $Q_{\epsilon_F}/Q_{\epsilon_U}$ indices with respect to the the static friction coefficient at the contacts μ in a fully actuated and underactuated power grasp with $n_c = 6$. The value of $\mathbf{f}_{max}$ is fixed to $30N$. (c), (d): PGR and $Q_{\epsilon_F}/Q_{\epsilon_U}$ indices with respect to the the maximum applicable force at the contacts f_{max} in a fully actuated and underactuated power grasp with $n_c = 6$. The value of μ is fixed to 0.8.

equal for all the contact points ($\mu_i = \mu, i = 1, \ldots, n_c$; $f_{i,max} = f_{max}, i = 1, \ldots, n_c$), and we choose their values according to [30].

A. Evaluation of Fully Actuated Grasps

Fig. 6 shows the results obtained when computing PGR and Q_{ϵ_F} with respect to n_c, μ, and f_{max} for a fully actuated hand. There are some characteristics that are common to all the graphs in Fig. 6: i) the quality is non-decreasing with an increase in the number of contact points n_c, ii) the quality does not increase substantially between four and five contacts grasps, iii) the quality improves considerably in the case of power grasp, iv) the quality is zero with two contacts grasp. The last characteristic comes from the fact that force closure can be achieved only with a minimum of three non-collinear contact points with hard finger contacts for 3D objects [23].

Although the values of the indices cannot be directly compared, it is interesting to notice that PGR and Q_{ϵ_F} show similar trends with respect to the considered parameters. From Figures 6a and 6b, it can be observed that the higher the friction coefficient, the more robust the grasp is. In fact, with higher values of μ it is easier to find contact forces that satisfy the friction constraints (1), (2), and so larger external wrench perturbations can be resisted. The measures were computed considering $f_{max} = 30N$. According to the graphs in Figures 6c and 6d, with higher applicable forces, higher quality grasps can be performed as well. In this case, μ was fixed to be 0.8.

B. Evaluation of Underactuated Grasps

As underlined in Section III and Section IV, both Q_{ϵ_U} and PGR measures can account for the underactuation of the hand. In this section, we investigate whether they have similar trends to those in Fig. 6 while considering underactuation. In particular, we compute the indices for a power grasp with six contacts (Fig. 3b), as underactuated hands are most likely to perform power grasps.

For the evaluation of PGR the anthropomorphic hand model is actuated only with the first three postural synergies, which can describe the majority of the grasps performed by humans according to [25]. From Fig. 7a and Fig. 7c, it can be seen that the PGR in the case of underactuation is slightly lower, but has a similar trend compared to full actuation. Fig. 7b and Fig. 7d show that Q_{ϵ_U} in the case of underactuation is also slightly lower but has a similar trend compared to full actuation Q_{ϵ_F}. For the evaluation of Q_{ϵ_U} measure, the actuation matrix reflecting that two contacts are powered by the same actuator (Fig. 3b) is

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Results in Fig. 7 indicate that, even with the consideration of underactuation, PGR and Q_{ϵ_U} have similar trends, and they behave coherently with respect to the fully actuated hand: the quality increases when μ and f_{max} increase.

VI. Conclusions and Future Work

This paper extends existing concepts on grasp quality measures to the evaluation of grasps with underactuated hands. In particular, measures based on the wrench space computation and measures of contact and grasp robustness are compared through simulated examples. Despite their different approach to model underactuation, both types of indexes can account for friction constraints and physically achievable contact forces, and they are found to have similar trends in all the analyzed examples. These results confirm that Q_{ϵ_U} and PGR measures are suitable for evaluating underactuated grasps in a realistic and coherent way.

As a next step, the proposed measures will be applied to grasp planning and execution with real robotic hands, including experimental validation of the proposed measures. The presented measures assume a perfect knowledge of the location of the contact points; however, in experiments with soft and highly underactuated robotic hands it is not easily guaranteed that the optimal contact points specified by the planner or predicted by a simulator are perfectly reached. This can be due to factors such as the low number of control inputs, the deformability of the hand, and to uncertainties on shape and location of the object. The uncertainty in the contacts' position could be included in the computation of the quality measures, for instance extending the concept of Independent Contact Region (ICR) [18]. Other topics of future research include the development of a method to consider postural synergies in the computation of the wrench-space-based measures, and further development of the actuation matrix to better reflect different underactuation architectures: having a common and general way to model underactuation will facilitate the experimental computation of the proposed measures for real robotic grasps.

References

[1] R. Deimel and O. Brock, "A novel type of compliant and underactuated robotic hand for dexterous grasping," *Int. J. Robotics Research*, vol. 35, no. 1-3, pp. 161–185, 2016.

[2] M. Ciocarlie and P. Allen, "A design and analysis tool for underactuated compliant hands," in *Proc. IEEE/RSJ Int. Conf. on Intelligent Robots and Systems - IROS*, 2009, pp. 5234–5239.

[3] C. D. Santina, G. Grioli, M. Catalano, A. Brando, and A. Bicchi, "Dexterity augmentation on a synergistic hand: The Pisa/IIT Softhand+," in *Proc. IEEE-RAS Int. Conf. on Humanoid Robots*, 2015, pp. 497–503.

[4] A. Bicchi, "On the closure properties of robotic grasping," *The Int. J. of Robotics Research*, vol. 14, no. 4, pp. 319–334, 1995.

[5] M. A. Roa and R. Suárez, "Grasp quality measures: Review and performance," *Autonomous Robots*, vol. 38, no. 1, pp. 65–88, 2015.

[6] J. Tegin, B. Iliev, A. Skoglund, D. Kragic, and J. Wikander, "Real life grasping using an under-actuated robot hand-simulation and experiments," in *Proc. IEEE Int. Conf. Advanced Robotics - ICAR*, 2009, pp. 1–8.

[7] D. M. Aukes and M. R. Cutkosky, "Simulation-based tools for evaluating underactuated hand designs," in *Proc. IEEE Int. Conf. Robotics and Automation - ICRA*, 2013, pp. 2067–2073.

[8] R. Balasubramanian and A. M. Dollar, "Performance of serial underactuated mechanisms: Number of degrees of freedom and actuators," in *Proc. IEEE/RSJ Int. Conf. on Intelligent Robots and Systems - IROS*, 2011, pp. 1823–1829.

[9] G. Kragten, A. Kool, and J. Herder, "Ability to hold grasped objects by underactuated hands: performance prediction and experiments," in *Proc. IEEE Int. Conf. Robotics and Automation - ICRA*, 2009, pp. 2493–2498.

[10] N. Pollard , "Closure and quality equivalence for efficient synthesis of grasps from examples," *Int. J. Robotics Research*, vol. 23, no. 6, pp. 595–614, 2004.

[11] D. Prattichizzo, J. K. Salisbury, and A. Bicchi, "Contact and grasp robustness measures: Analysis and experiments," in *Experimental Robotics-IV*, ser. Lecture Notes in Control and Information Science 223. Springer, 1997.

[12] D. Prattichizzo, M. Malvezzi, M. Gabiccini, and A. Bicchi, "On motion and force controllability of precision grasps with hands actuated by soft synergies," *IEEE Trans. Robotics*, vol. 29, no. 6, pp. 1440–1456, 2013.

[13] C. Ferrari and J. Canny, "Planning optimal grasps," in *Proc. IEEE Int. Conf. Robotics and Automation - ICRA*, 1992, pp. 2290–2295.

[14] J. Bohg, A. Morales, T. Asfour, and D. Kragic, "Data-driven grasp synthesis - a survey," *IEEE Trans. Robotics*, vol. 30, no. 2, pp. 289–309, 2014.

[15] R. M. Murray, Z. Li, and S. S. Sastry, *A mathematical introduction to robotic manipulation*. CRC press, 1994.

[16] Y. Zheng and W.-H. Qian, "Limiting and minimizing the contact forces in multifingered grasping," *Mechanism and Machine Theory*, vol. 41, no. 10, pp. 1243–1257, 2006.

[17] C. Borst, M. Fischer, and G. Hirzinger, "Efficient and precise grasp planning for real world objects," in *Multi-point interaction with real and virtual objects*. Springer, 2005, pp. 91–111.

[18] M. A. Roa and R. Suarez, "Computation of independent contact regions for grasping 3-D objects," *IEEE Transactions on Robotics*, vol. 25, no. 4, pp. 839–850, Aug 2009.

[19] K. Hertkorn, M. A. Roa, T. Wimbock, and C. Borst, "Simultaneous and realistic contact and force planning in grasping," in *Proc. IEEE/RSJ Int. Conf. on Intelligent Robots and Systems - IROS*, 2015, pp. 2291–2298.

[20] H. Jeong and J. Cheong, "Evaluation of 3D grasps with physical interpretations using object wrench space," *Robotica*, vol. 30, no. 3, pp. 405–417, 2012.

[21] Y. Zheng and K. Yamane, "Evaluation of grasp force efficiency considering hand configuration and using novel generalized penetration distance algorithm," in *Proc. IEEE Int. Conf. Robotics and Automation - ICRA*, 2013, pp. 1580–1587.

[22] M. Cutkosky and I. Kao, "Computing and controlling compliance of a robotic hand," *IEEE Trans. Robotics and Automation*, vol. 5, no. 2, pp. 151–165, 1989.

[23] D. Prattichizzo and J. Trinkle, "Grasping," in *Handbook on Robotics*. Springer, 2008, pp. 671–700.

[24] A. Bicchi, "On the problem of decomposing grasp and manipulation forces in multiple whole-limb manipulation," *Robotics and Autonomous Systems*, vol. 13, no. 2, pp. 127 – 147, 1994.

[25] M. Santello, M. Flanders, and J. F. Soechting, "Postural hand synergies for tool use," *Journal of Neuroscience*, vol. 18, no. 23, pp. 10 105–10 115, 1998.

[26] G. Gioioso, G. Salvietti, M. Malvezzi, and D. Prattichizzo, "Mapping synergies from human to robotic hands with dissimilar kinematics: an approach in the object domain," *IEEE Trans. Robotics*, vol. 29, no. 4, pp. 825–837, 2013.

[27] M. Malvezzi, G. Gioioso, G. Salvietti, and D. Prattichizzo, "Syngrasp: A matlab toolbox for underactuated and compliant hands," *IEEE Robotics and Automation Magazine*, vol. 22, no. 4, pp. 52–68, 2015.

[28] G. Baud-Bovy, D. Prattichizzo, and N. Brogi, "Does torque minimization yield a stable human grasp?" in *Multi-Point Physical Interaction with Real and Virtual Objects*, ser. STAR, 2005, vol. 18, pp. 21–40.

[29] M. R. Tremblay and M. R. Cutkosky, "Estimating friction using incipient slip sensing during a manipulation task," in *Proc. IEEE Int. Conf. Robotics and Automation - ICRA*, 1993, pp. 429–434.

[30] M. Gabiccini, A. Bicchi, D. Prattichizzo, and M. Malvezzi, "On the role of hand synergies in the optimal choice of grasping forces," *Autonomous Robots*, vol. 31, pp. 235–252, 2011.